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A REINTERPRETATION OF THE PALMGREN-MINER RULE FOR FATIGUE LIFE PREDICTION

BY

ZVI HASHIN

DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING

COLLEGE OF ENGINEERING AND APPLIED SCIENCE

UNIVERSITY OF PENNSYLVANIA

PHILADELPHIA, PENNSYLVANIA

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*Visiting Professor -- Department of Materials Science and Engineering, University of Pennsylvania, Philadelphia, Pennsylvania 19104, U.S.A.

1. Introduction

The simplest and most well known assumption for determination of fatigue lifetime under variable amplitude cyclic loading is due to Palmgren [1] and Miner [2]. Mathematically it is expressed by the statement

$$\sum^{k} \frac{n_k}{N(\sigma_k)} = 1 \tag{1.1}$$

where the amplitudes of cycling are piecewise constant with k^{th} amplitude σ_k, and number of cycles n_k, and $N(\sigma_k)$ is the lifetime for constant amplitude σ_k cycling.

Equ. (1.1) is easily interpreted for the case where the cyclic amplitude is a continuous function $\sigma(\bar{n})$ of the number of elapsed cycles $\bar{n}$. In that case (1.1) becomes

$$\int_{\sigma_0}^{\sigma_u} \frac{d\bar{n}(\sigma)}{N(\sigma)} = 1 \tag{1.2}$$

where $\bar{n}(\sigma)$ is the inverse of $\sigma(\bar{n})$, σ_0 is the initial amplitude and σ_u is the amplitude at failure. The lifetime $\bar{n}_u$ is then given by

$$\bar{n}_u = \bar{n}(\sigma_u) \tag{1.3}$$

provided that (1.3) is a single valued function.

In the following the assumption (1.1) shall be referred to as the PM rule. It has first been stated in [1] and has been reintroduced in [2] on the basis of some physical assumptions. Other interpretations in terms of assumed crack growth regimes have been given in the literature. See e.g. [3]. The chief criticisms of (1.1)

are that it ignores sequence of loading effects and that the only material information used is the S-N curve, thus fatigue failure information under constant amplitude only. However, the PM rule does agree with experimental data in certain cases but is also very inaccurate in others.

It is the purpose of the present work to show that (1.1) is a very special case of a general theory of fatigue lifetime prediction, i.e., cumulative damage theory, and to examine some resulting consequences.

It should, however, be emphasized that the theories of cumulative damage here considered are of deterministic nature. In comparison of the results of such theories to the usual significantly scattered test results there arises the fundamental question: What is the experimental interpretation of a deterministically predicted lifetime? Is it the average of the scattered lifetimes under identical cyclic loading programs or is it some other associated statistical parameter? To the writer's knowledge a satisfactory answer to these questions is not available.

2. Resumé of Cumulative Damage Theory

For present purposes it is necessary to give a brief summary of the cumulative damage theory developed in [4] for prediction of fatigue lifetime under general cyclic loading programs.

It is assumed that there is available a family of "identical" specimens. Each of these specimens is cycled to failure in a two stage loading. The first stages of all of the two stage loadings are identically n_1 cycles at stress amplitude σ_1. The second stages are at different amplitudes σ_j with residual lifetimes $n_j{}^r$. The

n_j^r are plotted starting from the S-N curve of the material horizontally to the left at ordinates σ_j. The resulting locus is defined as the *damage curve* through n_1, σ_1, fig. 1.

The following properties of damage curves have been established in [4]:

1. A damage curve is uniquely defined by one of its points. This is based on an equivalent loading postulate which will be explained further below.

2. All damage curves pass through the static ultimate point 0, σ_s.

3. Damage curves do not intersect the n or σ axes, except at 0, σ_s.

4. Damage curves do not intersect (if the material is such that additional cycling reduces the residual lifetime) except at static ultimate point and fatigue limit.

It is thus seen that the damage curves form a family of curves which cover the region bounded by the n,σ axes and the S-N curve. The actual shape of the damage curves is not known. But since the S-N curve is a special case of the damage curves (one stage loading; residual lifetime in second stage vanishes) it is perhaps not unreasonable to assume that the damage curves are expressed by a similar mathematical form as the S-N curve, [4]. Thus if the S-N curve is represented as

$$s = f(\Gamma, n) \qquad (2.1)$$

where s is a nondimensional stress σ/σ_s and Γ is a curve fitting parameter, then the damage curves are represented by the one parameter family

$$s = f(\gamma, n) \tag{2.2}$$

where γ is a parameter which changes from curve to curve. It is of course possible to represent damage curves by a family with more than one parameter.

Many S-N curves can be adequately represented by straight lines in semi-log or log-log coordinates. Thus

$$\left.\begin{array}{lll} s = 1 + \Gamma \log n & \text{semi-log} & \text{(a)} \\ \log s = \Gamma \log n & \text{log-log} & \text{(b)} \end{array}\right\} \tag{2.3}$$

In that case it follows from (2.1.2) that the damage curve families have the forms

$$\left.\begin{array}{ll} s = 1 + \gamma \log n & \text{(a)} \\ \log s = \gamma \log n & \text{(b)} \end{array}\right\} \tag{2.4}$$

where γ is determined by the coordinates of any point on the damage curve.

The behavior of the damage curves in the neighborhood of a fatigue limit is not clear at the present time. Let the fatigue limit stress be denoted σ_e and consider two stage loadings with first stage $\sigma_1 > \sigma_e$ for n_1 cycles and second stage σ_e. If a specimen is cycled at constant level σ_e the lifetime will, by definition of the fatigue limit, be infinite (that is, longer than maximum acceptable cycling time). For the two stage loading described it is quite possible to have finite lifetime at σ_e which is more-

over a decreasing function of σ_1 and n_1. It is also reasonable to assume that for any High-Low two stage loading, with first stage σ_1 for n_1 cycles, there will exist a fatigue limit $\bar{\sigma}_e(\sigma_1, n_1)$ which is a decreasing function of σ_1 and n_1. If there is made the simplifying assumption that $\bar{\sigma}_e \sim \sigma_e$ then all damage curves which represent the two stage loading described above must terminate at stress level σ_e. If the S-N curve is a straight line in semi-log or log-log coordinates, the fatigue limit is represented by a break in the S-N straight line, fig. 2. If the damage curves are assumed linear in semi-log or log-log representation then they must converge into the fatigue limit point $n = N_e$; $s_e = \sigma_e/\sigma_s$, fig. 2. The equations of these straight lines are given by

$$\begin{aligned} s - s_e &= \gamma \log\left(\frac{n}{N_e}\right) \qquad \text{semi-log} \quad \text{(a)} \\ \log(s/s_e) &= \gamma \log\left(\frac{n}{N_e}\right) \qquad \text{log-log} \quad \text{(b)} \end{aligned} \qquad (2.5)$$

It is to be expected that (2.4) will approximate the damage curves in the neighborhood of the static ultimate point $n=0$; $s=1$ while (2.5) will approximate them in the neighborhood of the fatigue limit. It is of course possible to construct nonlinear damage curves which will be tangent to the two sets of straight lines at static ultimate and fatigue limit but this subject will not be considered here.

The damage curves determine, by definition, lifetimes under all two stage loadings. It has been shown in [4] that lifetime under any cyclic loading program can be determined on the basis of the damage curves if it is assumed that specimens obey an equivalent loading postulate. To explain this postulate it is first necessary

to define equivalent cyclic loadings: Consider any variable amplitude loading program which terminates before failure occurs. Subsequently, the specimen is subjected to constant amplitude cycling-to-failure at some stress level, s_1. The residual lifetime under s_1 cycling is $n^r(s_1)$. Among the infinity of possible loading programs there must necessarily be some which have the same $n^r(s_1)$. Such loading programs are defined as equivalent cyclic loadings with respect to s_1. In conventional terms, equivalent loading means that specimens have "suffered the same amount of damage". This vague statement has, here, been precisely expressed in terms of equal residual lifetime under subsequent constant amplitude loading.

Since the stress level, s_1, is arbitrary it is reasonable to believe that if the subsequent constant amplitude is s instead of s_1, residual lifetimes will be the same $n^r(s) \neq n^r(s_1)$ for the cyclic loading programs equivalent with respect to s_1. Hence the *equivalent loading postulate* is stated as: *cyclic loadings which are equivalent for one stress level are equivalent for all stress levels.*

This postulate is schematically illustrated in fig. 3. The plots show variations of nondimensional amplitude of cyclic loadings. The loadings are equivalent for amplitude s_1 since $n^r(s_1)$ are the same. The equivalent loading postulate then asserts that they are equivalent for s_2 implying that n_2^r are the same and similarly for any other constant s cycling.

Residual lifetimes in two stage loadings are determined by the damage curves, in view of their definition. For a piecewise constant amplitude (multistage) loading the analysis procedure for residual lifetime is shown in fig. 4. First loading stage, n_1 cycles at s_1 amplitude, is traced in the S-N plane by the horizontal

segment n_1 at s_1. For next stage, n_2 cycles at s_2 amplitude, proceed on damage curve through n_1, s_1 until level s_2 and then advance n_2 horizontally. This is repeated for the various loading stages until the S-N curve is reached. The sum of the n_1, n_2, ... then defines the lifetime under the multistage loading program.

Now let the cyclic loading have a continuously variable amplitude defined by

$$
\begin{aligned}
s &= s(\bar{n}) \\
\bar{n} &= \bar{n}(s)
\end{aligned}
\tag{2.6}
$$

The damage curve equation (2.2) is written in the alternative forms

$$
\begin{aligned}
n &= g(\gamma, s) \\
\gamma &= \gamma(n, s)
\end{aligned}
\tag{2.7}
$$

In order to find the amplitude s_u at which failure occurs under the cyclic loading (2.6) it is necessary to solve the differential equation

$$
\frac{dn}{ds} = \left. \frac{\partial(\gamma, s)}{\partial s} \right|_{\gamma=\gamma(n, s)} + \bar{n}'(s) \tag{2.8}
$$

with initial condition

$$
s(0) = s_0 \tag{2.9}
$$

where s_0 is the initial amplitude of (2.6). The solution of (2.8) defines a curve s(n) in the s-n plane which intersects the S-N curve at failure amplitude s_u. The lifetime $\bar{n}_u$ is then given from (2.6.b) by

$$n_u = \bar{n}(s_u) \tag{2.10}$$

All of this presupposes that (2.6) are single valued functions. If this is not the case the multivalued function must be separated into single valued branches and the integrations must be carried out separately and successively for the various branches.

Various cases of multistage and continuous cyclic loadings have been treated in [4] on the basis of the damage curves (2.4) and (2.5).

3. Palmgren-Miner Cumulative Damage

The PM assumption will now be examined on the basis of the general cumulative damage theory summarized above. To construct the damage curves let a specimen be subjected to a two stage loading n_1 cycles at amplitude s_1 and then n^r cycles to failure at amplitude s. According to the PM assumption

$$\frac{n_1}{N_1} + \frac{n^r}{N_1} = 1 \tag{3.1}$$

and from fig. 1

$$n^r = N - n \tag{3.2}$$

Combination of (3.1) and (3.2) yields

$$\frac{n}{N} = \frac{n_1}{N} = \text{const} = \gamma \tag{3.3}$$

Thus the equation of the damage curves is

$$n(s) = \gamma\ N(s) \qquad 0 \leq \gamma \leq 1 \tag{3.4}$$

where N(s) is the equation of the S-N curve, $\gamma=1$ corresponds to the

S-N curve and $\gamma=0$ corresponds to the s axis. The value of γ for any damage curve is determined in terms of the coordinates of a point through which it passes by (3.3).

Two stage loadings do not in general obey the PM assumption. The following trends have been observed in metal fatigue

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} \begin{cases} > 1 \text{ when } s_1 < s_2 & \text{Low-High} \\ < 1 \text{ when } s_1 > s_2 & \text{High-Low} \end{cases} \tag{3.5}$$

The condition (3.5) has interesting implications for the damage curves. Writing the left side of (3.5) in terms of the substitution (3.2) for n_2 it follows at once that

$$\frac{n(s_1)}{N(s_1)} > \frac{n(s_2)}{N(s_2)} \tag{3.6}$$

In words: the necessary and sufficient condition to fulfill (3.5) is for $n(s)/N(s)$ to be a monotonically decreasing function of s. In differential form

$$\frac{d}{ds}\frac{n(s)}{N(s)} < 0$$

which can be reduced to

$$\frac{d}{ds}[\log n(s)] < \frac{d}{ds}[\log N(s)] \tag{3.7}$$

If the inequalities (3.5) are reversed, which has been found to be the case in some fiber composite testing, then the inequalities (3.6) and (3.7) also reverse. (3.6-7) become equalities if, and only if, the PM assumption is valid. It is easily seen that the logarithmic linear damage curves (2.4) obey inequalities in the opposite sense to

(3.5) and while the other kind, which converge into the fatigue limit, obey (3.5). Indeed it has been found, [4], that two stage fatigue life tests for steel are in good agreement with predictions based on (2.5).

Next the case of multistage loadings is considered. Referring to fig. 4 the damage curves needed are numbered consecutively. The abscissa of a point with ordinate s_j on the i^{th} damage curve will be denoted n_{ij}. Suppose the loading consists of the three stages n_1 cycles at amplitude s_1, n_2 at s_2 and n_3 to failure at s_3. It is required to find s_3. The procedure is indicated in fig. 4. The equation of damage curve 1 is

$$\frac{n}{N} = \frac{n_1}{N_1}$$

It follows that

$$n_{12} = \frac{n_1}{N_1} N(s_2) = \frac{n_1}{N_1} N_2 \qquad n_{22} = \frac{n_1}{N_1} N_2 + n_2$$

The equation of damage curve 2 is

$$\frac{n}{N} = \frac{n_{22}}{N_2}$$

It follows that

$$n_{23} = \frac{n_{22}}{N_2} N(s_3) = \frac{n_{22}}{N_2} N_3 \qquad n_{33} = n_{23} + n_3$$

Failure at amplitude s_3 cycling occurs when the S-N curve is reached at that level, i.e., when

$$n_{33} = N_3 \tag{3.8}$$

Combining (3.8) with the preceding relations it follows easily that

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

which is the PM assumption for a three stage loading. It is not difficult to show (by induction) that by this procedure failure in a general multistage loading will be predicted by (1.1).

It should be carefully noted that the PM assumption has been here adopted only for two stage loadings, whereby the form of the damage curves was determined. The procedure for multistage loading analysis in terms of the damage curves is based on the equivalent loading postulate which refers to any damage curves.

The case of continuous amplitude variation is governed by the differential equation (2.9). Analytical integration of this equation does not seem feasible in general. Even with the simple damage curves (2.4) integration could only be carried out numerically, [4]. The situation is however different in the case of the PM damage curves (3.4). In the present case (2.7) are given by (3.4) and (3.3). It follows that (2.8) assumes the form

$$\frac{dn}{ds} = \frac{n\, N'(s)}{N(s)} + \bar{n}'(s) \tag{3.9}$$

where a prime denotes differentiation. It is easily verified that the solution of (3.9) is

$$n(s) = N(s) \int_{s_0}^{s} \frac{\bar{n}'(\alpha)d\alpha}{N(\alpha)} \tag{3.10}$$

Failure is defined by the intersection of n(s) with the S-N curve N(s), i.e., when n(s) = N(s). Thus for failure

$$\int_{s_0}^{s_u} \frac{\bar{n}'(\alpha)d\alpha}{N(\alpha)} = 1 \qquad (3.11)$$

It is easily seen that (3.11) is the same as (1.2), the continuous amplitude variation version of the PM assumption.

The left sides of (1.1) and (1.2) may be termed the PM coefficient. Recall the inequalities (3.5) for this coefficient for two stage loading and their relations (3.6-7) to the damage curves. It is not difficult to show that (3.6-7) also imply that the general PM coefficient obeys inequalities (3.5) and also reversed inequalities (3.6-7) for reversed inequalities (3.5).

Experience accumulated over many years has shown that the PM rule sometimes predicts fatigue life with sufficient engineering accuracy while at other times it is very much in error. To the writer's knowledge no criteria for acceptability or inacceptibility of this rule are available.

In the course of present research, concerning the new cumulative damage theory, fatigue life predictions have been performed for various cyclic loadings on the basis of assumed linear damage curves (2.4) and (2.5). It has been found that (2.5) are more appropriate for metals. For two stage loadings there results the simple formula

$$\left(\frac{n_1}{N_1}\right)^{\log(N_2/N_e)/\log(N_1/N_e)} + \frac{n_2}{N_2} = 1 \qquad (3.12)$$

where N_e is lifetime at s_e, the fatigue limit, as determined by the S-N curve. It has been found [4] that (3.12) is in much better agreement with steel test data than the PM rule.

It is of considerable interest to have some general assessment of the expected differences between lifetime prediction as given by the PM rule and the much more general cumulative damage theory of [4]. While such general assessment is not available at the present time the following example will perhaps serve to establish trends.

Consider a multistage loading program composed of m stages, n_k cycles at amplitude s_k. The amplitudes increase or decrease monotonically in the interval from initial value s_1 to final value s_m, fig. 5.

A procedure for lifetime prediction under multistage loading has been given in [4]. This will here be summarized in a modified form. Define the recurrence relations

$$\mu_1 = \frac{n_1}{N_1}$$

$$\mu_2 = \mu_1^{\phi_2/\phi_1} + \frac{n_2}{N_2}$$

$$\vdots$$

$$\mu_k = \mu_{k-1}^{\phi_k/\phi_{k-1}} + \frac{n_k}{N_k} \tag{3.13}$$

where

n_k -- the number of cycles with amplitudes s_k in k^{th} stage,

$N_k = N(s_k)$ -- lifetime at s_k from S-N curve,

ϕ_k -- functions of stress amplitude defined by the form of damage curves.

Then failure is predicted by

$$\mu_k = 1 \tag{3.14}$$

Choosing the damage curves in the form (2.5a), that is semi-log linear through fatigue limit, the functions ϕ_k assume the form

$$\phi_k = s_k - s_e \tag{3.15}$$

It is noted in passing that for damage curves (2.5b)

$$\phi_k = \log(s_k/s_e) \tag{3.16}$$

If s_k in (3.15-16) are expressed in terms of N_k from the S-N curves (2.3) then in both cases

$$\frac{\phi_k}{\phi_{k-1}} = \frac{\log(N_k/N_e)}{\log(N_{k-1}/N_e)} \tag{3.17}$$

Equ. (3.12) for two stage loading is a special case of (3.13) with (3.17).

It is easily seen that the PM rule is obtained as a special case of (3.13-14) when ϕ_k=const.

Numerical computations have been carried out for ascending stair case loadings and their descending reverses, with initial and final amplitudes

$$s_1 = 0.3 \qquad s_m = 0.7$$
$$s_1 = 0.7 \qquad s_m = 0.3$$

In all cases

$$s_e = 0.2 \qquad \Gamma = -0.1$$

where Γ refers to S-N curve (2.3a), and

$$s_k - s_{k-1} = \Delta s = \text{const} = \frac{s_m - s_1}{m} \tag{3.18}$$

In the first set of computations it was assumed that $n_k=n=\text{const.}$ If a number of stages m is chosen the number of cycles n per stage then becomes the unknown to be determined from (3.13-15). It was found that for these cases the predictions agreed closely with the PM rule. Therefore, introducing (3.18) into (1.1) a good approximation is

$$n_u = 1 \Big/ \sum_{k=1}^{k=m} \frac{1}{N_k} \tag{3.19}$$

In the second set of computations it was assumed that

$$n_k = \beta N_k \tag{3.20}$$

where β = const. In this case the unknown for given number of stages is β, which is found numerically by satisfaction of (3.14). Once β is known the lifetime under given program is the sum of βN_k over all stress values.

The results of the computations are shown in fig. 6 as ratio of lifetime defined by (3.20) to lifetime predicted by the PM rule. Note that according to the latter

$$\beta_{PM} = \frac{1}{m}$$

It easily follows that the ratio plotted in fig. 6 is simply

$$\frac{\beta}{\beta_{PM}} = m\beta$$

It is seen that there is substantial disagreement with prediction of the PM rule. Also loading sequence reversal produces substantial changes in lifetime as is seen by the difference between ascending

and descending loading program lifetimes.

The differences in agreement with the PM rule in the different loadings are not difficult to explain. In the first case the number of cycles is the same for each stress level. Therefore the damage produced at different stress levels is significant only for the high stress levels. Roughly speaking the loading is equivalent to constant cycling for a certain number of cycles at max stress level of the loading. However for such (constant amplitude) loading the PM rule is (trivially) valid.

In the second kind of loading the damages done at each stress level are of similar magnitudes since the number of cycles in each stage is proportional to the lifetime at the stage stress level. Thus there is no reason to expect validity of the PM rule.

On the basis of the foregoing it may be speculated that the PM rule will be adequate when the most of the "damage" is done at roughly the same stress level. For a rough assessment the PM coefficient (1.1) or (1.2) may be computed. If in (1.1) one of the terms is dominant (larger than .9, say) or if in (1.2) the major contribution to the integral comes from a narrow stress band then it may be surmised that the PM rule would be an adequate approximation.

4. Conclusion

It has been shown that the well known Palmgren-Miner linear cumulative damage rule is a special case of a general cumulative damage theory. In this respect it should be noted that according to present development it is necessary to assume validity of the PM rule only for two stage loadings. Its validity for multistage

loadings then follows from a general equivalent loading postulate which is assumed valid for any cumulative damage theory.

It has been shown by means of numerical examples that there are classes of multistage loadings for which the general cumulative damage theory and the PM rule are in close agreement while substantial disagreement is found for others.

It must be emphasized again that all theories included here are phenomenological and deterministic. The fit of any such theory to test data is obscured by the significant scatter observed. It is therefore most important to generalize the theory developed so as to take into account the scatter of lifetime test data.

ACKNOWLEDGEMENT

It is a pleasure to acknowledge helpful and stimulating discussions with Dr. Campbell Laird.

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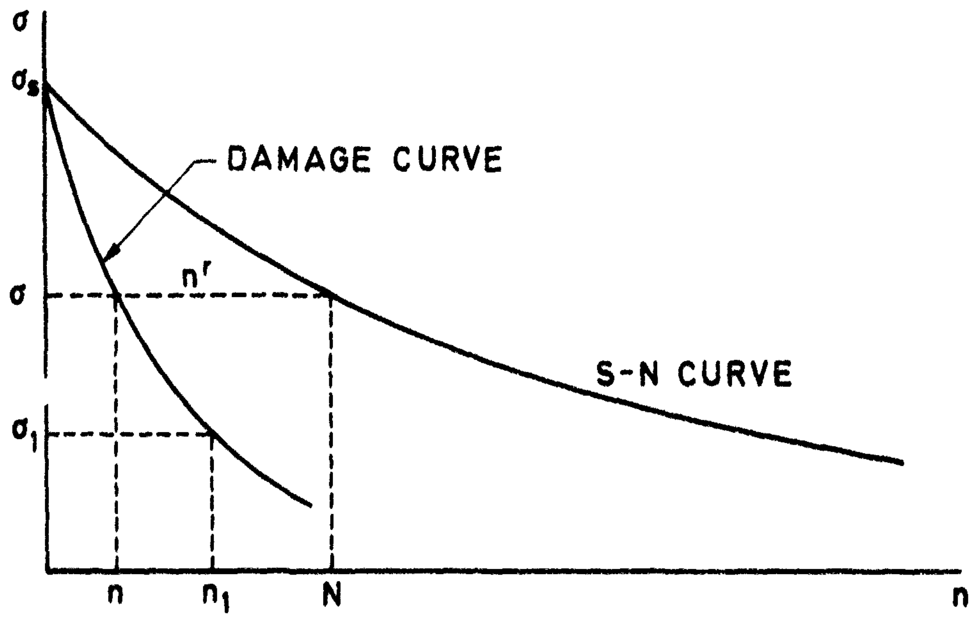

Figure 1 - Damage Curve

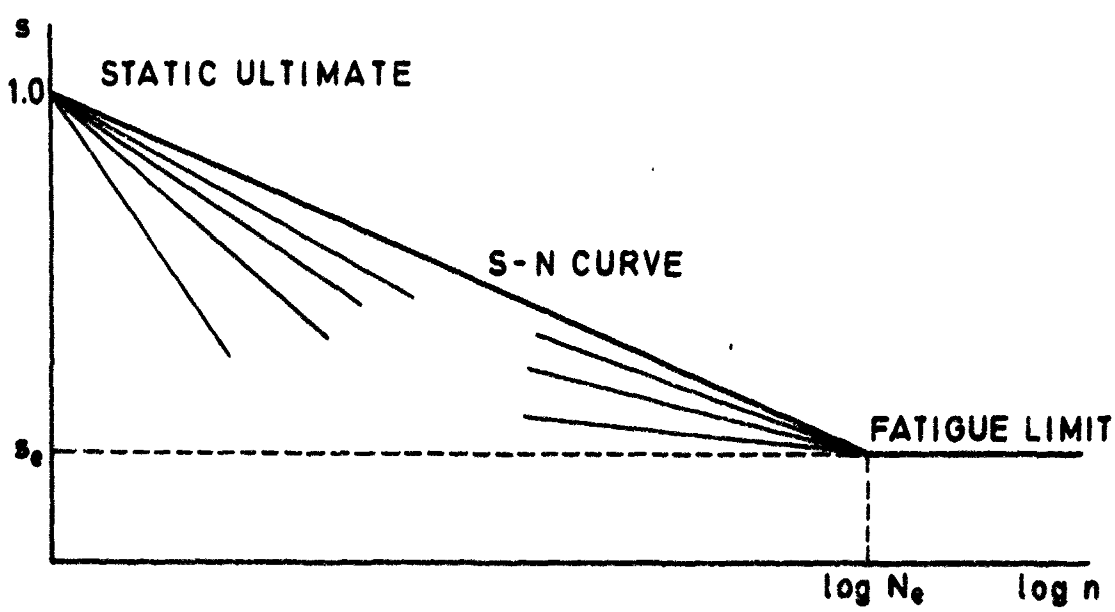

Figure 2 - Linear Damage Curves. Semi-log.

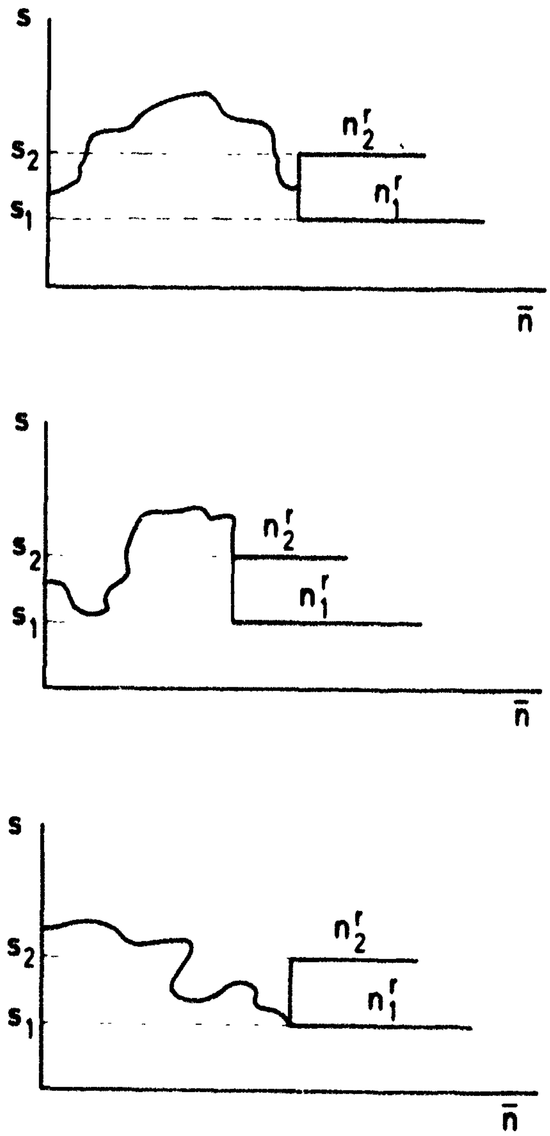

Figure 3 - Equivalent Loading Postulate

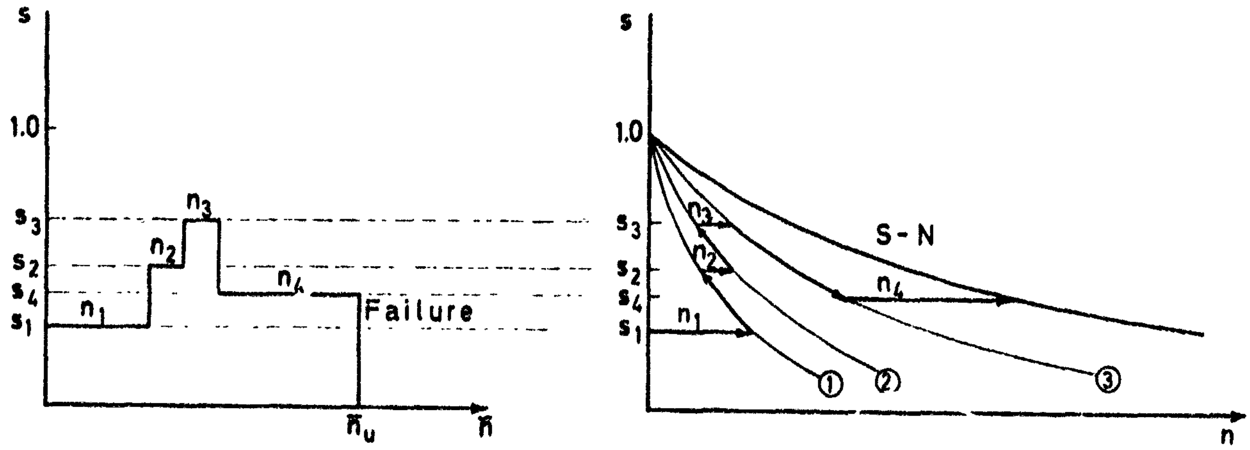

Figure 4 - Analysis of Multistage Loading

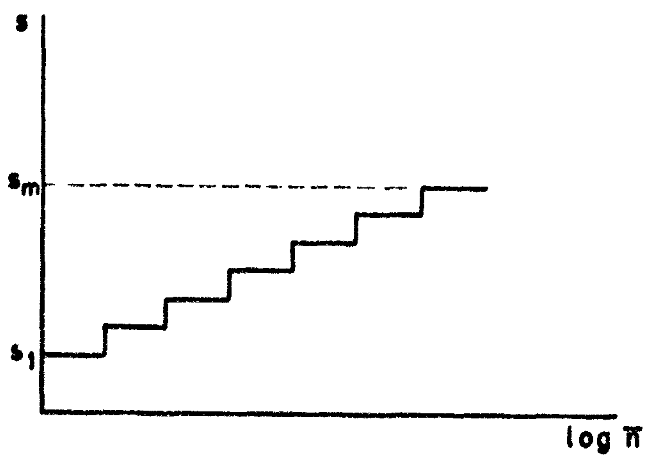

Figure 5 - Ascending Staircase Loading

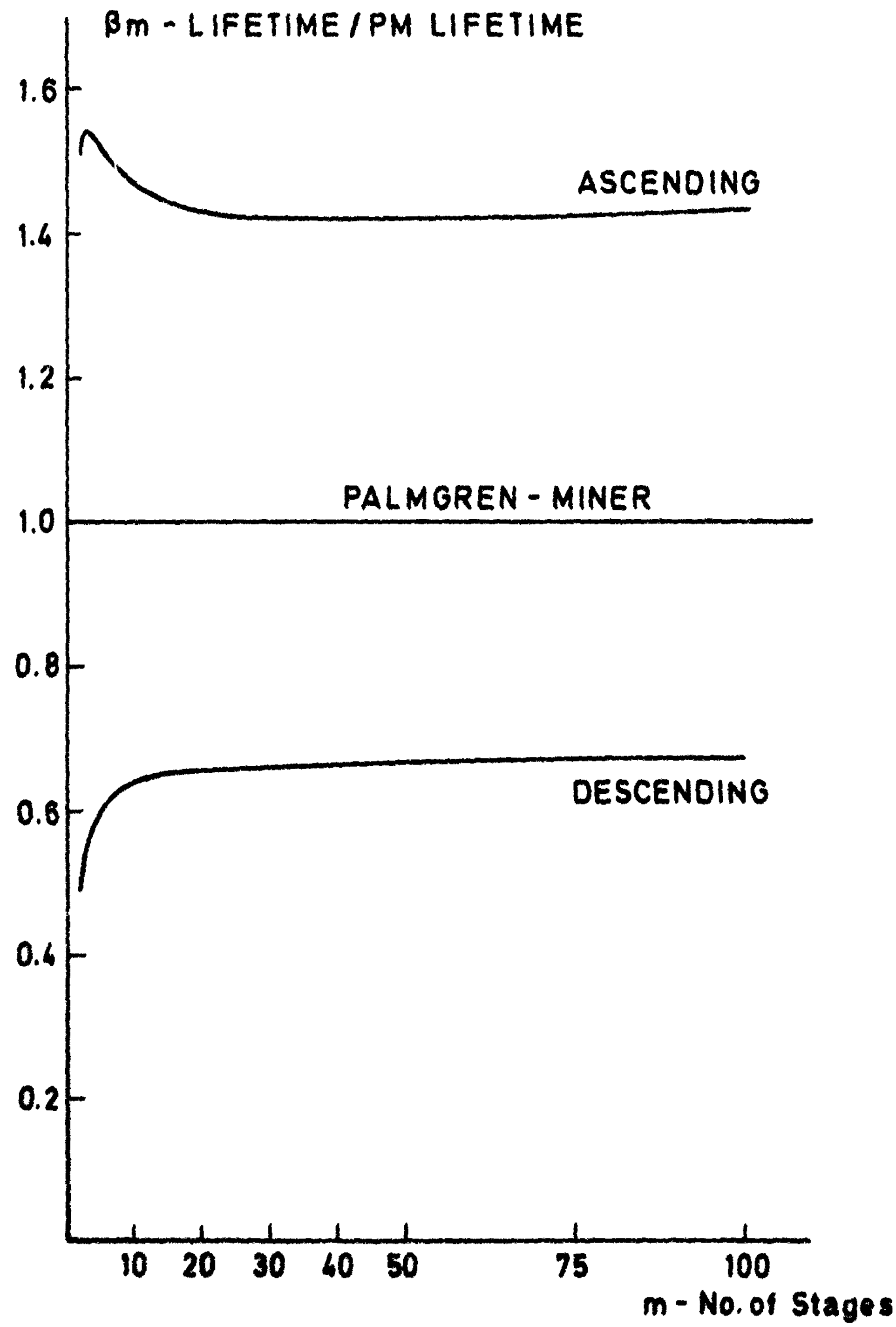

Figure 6 - Lifetimes for Staircase Loading Programs: Comparison With Palmgren-Miner Prediction.